예시 문제

Mathematics

Topics you should know:

1. Algebra: Binomial theorem, quadratic expressions, polynomials, theory of equations, complex numbers
2. Trigonometry: Trigonometric functions, identities, formulae and equations, relations between sides and angles of a triangle, inverse trigonometric functions.
3. Two-dimensional analytical/coordinate geometry: Cartesian coordinates, distance formula, locus, equation of a straight line in various forms, equation of a circle.
4. Differential calculus: Functions of a real variable, limits and continuity, derivatives, geometrical interpretation of the derivative, implicit differentiation, velocity and acceleration, tangents and normal.
5. Integral calculus: Indefinite and definite integrals, integration by parts, substitution, partial fractions, simple applications of definite integrals including areas under and enclosed by curves.

Two practice questions:

1. Consider the polynomial $P(x) = 4x^4 - 7x^3 + ax^2 + bx + 20$ with a and b real numbers such that $x - 1$ and $x + 2$ are factors of $P(x)$. Evaluate $P'(0)$.
   a. -29
   b. 12
   c. 17
   d. 29
   e. 34

Answer: b

The derivative of P is the function $P'(x) = 16\,x^3 - 21\,x^2 + 2ax + b$. Hence $P'(0) = b$. Since $x - 1$ is a factor of P, it holds that $P(1) = 0$ and thus that $4 - 7 + a + b + 20 = 0$. Similarly, $P(-2) = 0$ implies that $64 + 56 + 4a - 2b + 20 = 0$. To find the value of b and thus of $P'(0)$ we need to solve the following system of equations.

$$\begin{cases} a + b + 17 = 0 & (1) \\ 2a - b + 70 = 0 & (2) \end{cases}$$

From (2) we obtain $b = 2a + 70$. Substituting in (1) gives $3a = -87$ or $a = -29$. It follows that $b = 2(-29) + 70 = 12$.

예시 문제

2. Consider the function $f: \mathbb{R} \longrightarrow \mathbb{R}: x \mapsto x - \sqrt{x^2 + 5x}$. Which of the following statements is true?
   a. The graph of f has a horizontal asymptote when x tends to $+\infty$ and an oblique asymptote when x tends to $-\infty$.
   b. The graph of f has a horizontal asymptote when x tends to $-\infty$ and an oblique asymptote when x tends to $+\infty$.
   c. The graph of f has a horizontal asymptote both when x tends to $+\infty$ and when x tends to $-\infty$.
   d. The graph of f has an oblique asymptote both when x tends to $+\infty$ and when x tends to $-\infty$.
   e. The graph of f has no asymptotes.

Answer: a

The function has a horizontal asymptote $y = -\frac{5}{2}$ when x tends to $+\infty$. Indeed,

$$\lim_{x \to +\infty} x - \sqrt{x^2 + 5x} = \lim_{x \to +\infty} \frac{(x - \sqrt{x^2 + 5x})(x + \sqrt{x^2 + 5x})}{x + \sqrt{x^2 + 5x}} = \lim_{x \to +\infty} \frac{-5x}{x + \sqrt{x^2 + 5x}}$$

$$= \lim_{x \to +\infty} \frac{-5}{1 + \sqrt{1 + {}^{5}/_{x}}} = \frac{-5}{2}$$

The function has an oblique asymptote $y = 2x + \frac{5}{2}$ when x tends to $-\infty$. Indeed,

$$\lim_{x \to -\infty} x - \sqrt{x^2 + 5x} - 2x = \lim_{x \to -\infty} -x - \sqrt{x^2 + 5x} = \lim_{x \to -\infty} \frac{(-x - \sqrt{x^2 + 5x})(-x + \sqrt{x^2 + 5x})}{-x + \sqrt{x^2 + 5x}}$$

$$= \lim_{x \to -\infty} \frac{-5x}{-x + \sqrt{x^2 + 5x}} = \lim_{x \to -\infty} \frac{-5}{-1 - \sqrt{1 + 5/x}} = \frac{5}{2}$$

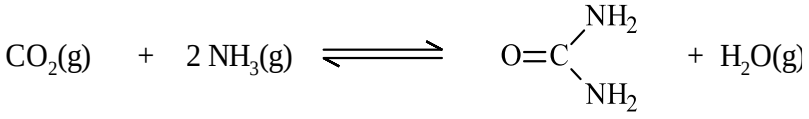
Chemistry

Topics you should know:

- Elements, mixtures
- Lavoisier Law
- Symbolic representation of atoms and molecules, atomic mass, unit of atomic mass, electrons and nucleons (protons and neutrons)
- Oxidation number, ion and ion charge
- Reactions between bases and acids
- Reaction equations: ion exchange reactions, precipitation reactions, combustion reactions, synthesis reactions
- pH calculations, titration and titration reactions
- Bohr atom model, Bohr-Sommerfeld model, electron spin, Pauli rule
- Energy levels: s, p, d, f and orbital (basic knowledge)
- Electro negativity, electron pairs
- Covalent and ionic bonds, metal bonds
- Lewis notation from binary compounds and polyatomic compounds
- Polar and apolar compounds
- Intermolecular forces
- Nomenclature inorganic and organic compounds and ions (basic level)
- Stoichiometry: molar mass, molar volume, Avogadro constant, ideal gas law, mass density
- Concentration and concentration units, calculation of masses, volumes, concentrations, excess and limiting reagentia
- Reaction rate: factors influencing reaction rate, explanation via collision theory model
- Chemical equilibrium: equilibrium constant, factors influencing chemical equilibrium, calculations with equilibrium data
- Redox reactions: completion of redox reactions and interpretation of oxidators and reductors Sigma and pi bonds
- Solubility of ionic compounds

Two practice questions:

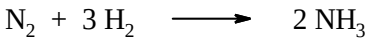
1. The German chemist Friedrich Wöhler discovered in 1828 that the carbon compound urea, $(NH_2)_2CO$, could be prepared from anorganic compounds. This discovery was very important for chemistry because it showed that natural compounds could be prepared in the chemical laboratory without using biological reagents. A possible synthesis of urea is



CO_2, NH_3 and H_2O are gaseous, urea is solid and the reaction is exothermic.
How can the temperature and pressure be adjusted so that the equilibrium shifts towards the side of urea?

(A) Increase temperature and decrease pressure
(B) Increase temperature and pressure
(C) Decrease temperature and pressure
(D) Decrease temperature and increase pressure
(E) Decrease temperature, a change in pressure does not affect the equilibrium

2. Ammonia is prepared according to the reaction



The reaction starts with a total pressure of 100 bar, nitrogen and hydrogen together, in a confined space at constant temperature. Initially the pressure of nitrogen is 62.5 bar and the pressure of hydrogen is 37.5 bar. What is the total pressure after completion of the reaction?

(A) 25 bar
(B) 50 bar
(C) 75 bar
(D) 100 bar
(E) 125 bar

Solution question 1

The reaction is exothermic, so Le Chatelier's principle can be adopted: ‘When a system at equilibrium is subjected to change in concentration, temperature, volume, or pressure, then the system readjusts itself to (partially) counteract the effect of the applied change and a new equilibrium is established.’ Hence, the temperature should be decreased.

Also, when there is an increase in pressure, the equilibrium will shift towards the side of the reaction with fewer moles of gas. Since urea is a solid, the reaction will proceed to the right in case of a pressure increase.

Final answer: a decrease in temperature and an increase in pressure will result in higher production of urea from carbon dioxide and ammonia.

Solution question 2

Total pressure is 100 bar, 62.5 bar for nitrogen and 37.5 bar for hydrogen. The stoichiometry of the reaction learns that 1 mole of nitrogen requires 3 moles of hydrogen. Since less hydrogen is present than nitrogen, the former is the so-called limiting reagent and will be consumed completely.

Hence, the hydrogen needs 37.5/3 = 12.5 bar of nitrogen, leaving 62.5 - 12.5 = 50 bar of nitrogen unused. In the given chemical reaction, 4 moles are converted into 2 moles so, via the ideal gas law (P*V = n*R*T), the pressure of 12.5 + 37.5 = 50 bar is reduced into 25 bar. Adding up gives a total pressure of 75 bar and the composition is 1/3 ammonia and 2/3 nitrogen.

Final answer: the total pressure, after completion of the reaction, is 75 bar.